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# EDGE-ODD GRACEFULNESS OF THE GRAPHS P2N , NP4 + 2PN+2 AND F2, N

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**Abstract - A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map $f_+$ :V(G) → {0, 1, 2, 3, …, (2k-1)} defined by $f_+(x) = \sum \{ f(x,y) / xy \in E\} \bmod 2k$ , where the vertex x is incident with other vertex y and k = max{p, q} makes all the edges distinct and odd. In this article, the Edge-odd gracefulness of the graphs $P_n^2$, $nP_4 + 2P_{n+2}$, and $F_{2,n}$ is obtained.**
**Key words - Graceful Graphs, Edge-odd graceful labeling, Edge-odd Graceful Graph**

## 1. INTRODUCTION AND BASIC DEFINITIONS

Graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labelings were first introduced in the late 1960s. In the intervening years dozens of graph labelings techniques have been studied in over 800 papers.

Graph labeling have often been motivated by practical problems, is one of fascinating areas of research. A systematic study of various applications of graph labeling is carried out in Bloom and Golomb[1]. Labeled graph plays vital role to determine optimal circuit layouts for computers and for the representation of compressed data structure.

The study of graceful graphs and graceful labeling methods was introduced by Rosa [2]. Rosa defined a valuation of a graph G with q edges as an injection from the vertices of G to the set{0, 1, 2, …, q} such that when each edge xy is assigned the label |f(x) - f(y)| , the resulting edge labels are distinct. Valuations are the functions that produce graceful labeling. However, the term graceful labeling was not used until Golomb studied such labeling several years later [3]. For a dynamic survey of various graph labeling problems along with an extensive bibliography we refer to Gallian [4].

We begin with simple, finite, connected and undirected graph G = (V, E) with p vertices and q edges. For all other standard terminology and notions we follow Harary [5].

Definition 1 : Graph

A graph is an ordered triple $G = (V(G), E(G), I_G)$, where $V(G)$ is a nonempty set, $E(G)$ is a set disjoint from $V(G)$, and $I_G$ is an incidence map that associates with each element of $E(G)$, an unordered pair of elements of $V(G)$.

Definition 2: The graph $P_n^2$

The graph $P_n^2$ is a connected graph with the combination of the path graphs $P_n$ and $P_2$.

Definition 3: The graph $nP_4 + 2P_{n+2}$

The graph $nP_4 + 2P_{n+2}$ is a connected graph of the combination of n times the path graph $P_4$ along with 2 times the path graph $P_{n+2}$.

Definition 4 : The graph $F_{2,n}$

A fan graph $F_{m,n}$ is defined as the graph join $\overline{K}_m + P_n$, where $\overline{K}_m$ is empty graph on m nodes and $P_n$ is the path graph on n nodes. The case m=1 corresponds to the usual fan graphs, while m=2 corresponds to the double fan, etc.

Definition 5: Graceful Graph

A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label │f(u) – f(v)│, the resulting edge labels are distinct. Then the graph G is graceful.

Definition 6: Edge Graceful Graph

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A graph G(V, E) is said to be edge-graceful if there exists a bijection f from E to {1, 2, …., |E|} such that the induced mapping $f_+$ from V to {0,1, …., |V|-1} given by $f_+(x) = \sum f(xy) \bmod |V|$ taken over all edges xy is a bijection.

Definition 7: Odd Graceful Graph

A graph G with q edges is to be odd graceful if there is an injection f:V(G)→{0, 1, 2,…,2q-1} such that, when each edge xy is assigned the label |f(x) - f(y)|, the resulting edge labels are {1, 3, …, 2q -1}.

Definition 8: Edge-odd graceful graph

A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map $f_+$:V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y) (\bmod 2k)$, where the vertex x is incident with other vertex y and k=max{p, q} makes all the edges distinct and odd. Hence the graph G is edge-odd graceful.

**2. MAIN RESULT**

In this section the edge odd graceful labeling of the graphs $P_n^2$, $nP_4 + 2P_{n+2,}$ and $F_{2,n}$ is presented.

Theorem 1: The graph $P_n^2$ for all n, but for n ≠ 8 is edge odd graceful.

Proof: The graph $P_n^2$ is a connected graph with n vertices and 2n-3 edges.

The arbitrary labeling of the graph $P_n^2$ is mentioned below.

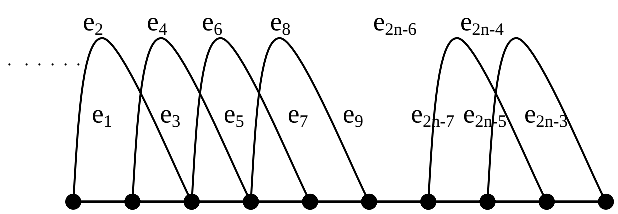


Figure 1 : Edge Odd graceful graph $P_n^2$

To find the edge odd graceful,

define f : E(G) → {1, 3, 5, . . . , 2q-1} by

f ($e_i$ ) = 2i-1, i = 1, 2, 3, . . . , 2n-3.

The above defined labelling function will induce the bijective vertex labeling function $f_+$ : V(G) → {0, 1, 2, ..., 2k-1} such that

$f_+(x) = \sum \{ f(x,y) / xy \in E \} \bmod 2k$ , where k = max {p, q}.

Hence the graph $P_n^2$ for n ≠ 8 is edge odd graceful.

Lemma 2: The graph $P_8^2$ is edge odd graceful.

Proof: The graph $P_8^2$ is a connected graph with 8 vertices and 13 edges.

The labeling of the graph is mentioned below.

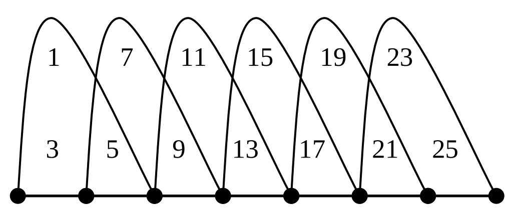


Figure 2: Edge Odd graceful graph $P_8^2$

To find edge odd graceful, define f : E(G) → {1, 3, 5, . . . , 2q-1} by

f ($e_i$ ) = 2i-1 , i = 1, 2, 3, . . . , 13

The above defined labeling function will induce the bijective vertex labelling function $f_+$ :V(G) → {0, 1, 2, . . . , 2k-1} such that $f_+(x) = \sum \{ f(x,y) / xy \in E \} \bmod 2k$ , where k = max {p, q}.

Hence the graph $P_8^2$ is edge odd graceful.

Theorem 3: The graph $nP_4 + 2P_{n+2}$ is edge odd graceful.

Proof: The graph $nP_4 + 2P_{n+2}$ is a connected graph with n vertices and 5n+2 edges.

The arbitrary labeling of the graph is given below.

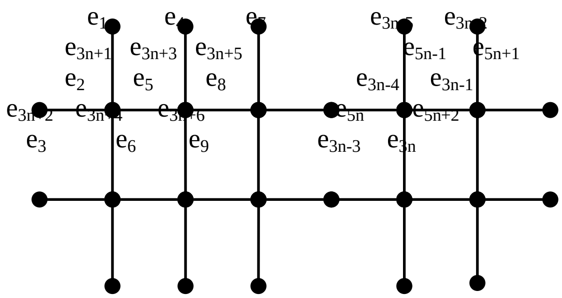


Figure 3: Edge Odd graceful graph $nP_4 + 2P_{n+2}$

To find edge odd graceful,
define f : E(G) → {1, 3, 5, . . . , 2q-1} by
$f(e_i) = 2i-1$ , $i = 1, 2, 3, \ldots, 5n+2$
The above defined labeling function will induce the bijective vertex labelling function $f_+ : V(G) \to \{0, 1, 2, \ldots, 2k-1\}$ such that $f_+(x) = \sum\{ f(x,y) / xy \in E\} \bmod 2k$ , where k = max {p, q}.
Hence the graph $nP_4 + 2P_{n+2}$ is edge odd graceful.
Theorem 4: The graph $F_{2,n}$ is edge odd graceful.
Proof: The graph $F_{2,n}$ is a connected graph with n+2 vertices and 3n-1 edges.
The arbitrary labeling of the graph is mentioned below.

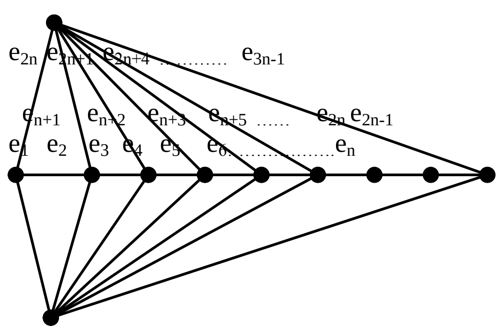


Figure 4: Edge Odd graceful graph $F_{2,n}$

To find edge odd graceful,
define f : E(G) → {1, 3, 5, . . . , 2q-1} as follows.

case (1) : $n \equiv 0 \pmod 4$

$$f(e_i) = 2i-1, \; i = 1, 2, 3, \ldots, 3n-1$$

case (2) : $n \equiv 1 \pmod 4$

$$f(e_i) = 2i-1, \; i = 5, 6, 7, \ldots, 3n-1$$

$$f(e_1) = 3, \; f(e_2) = 1, \; f(e_3) = 7, \; f(e_4) = 5$$

case (3) : $n \equiv 2 \pmod 4$

$$f(e_i) = 2i-1, \; i = 1, 2, 3, \ldots, 3n-1$$

case (4) : $n \equiv 3 \pmod 4$

$$f(e_i) = 2i-1, \; i = 1, 3, \ldots, n, n+2, \ldots, 3n-1 \quad f(e_2) = 2n+1, \quad f(e_{n+1}) = 3$$

The above defined labeling function will induce the bijective vertex labelling function $f_+$:V(G) → {0, 1, 2, . . . , 2k-1} such that $f_+(x) = \sum\{ f(x, y) / xy \in E\} \bmod 2k$ , where k = max {p, q} is distinct.

Hence the graph $F_{2,n}$ is edge odd graceful.

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